Bread Spin: Towards a Self-Consistent Theory of Sandwich Identification

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A precise mathematical structure is imposed on a system to distinguish foods based on starch placement. It is shown that starch foods are neatly represented as vectors in a Hilbert space with the relevant symmetries to be described by spinors. The theoretical framework is established and several immediate implications are discussed.

I. INTRODUCTION

Among the conversations dominating our cultural landscape (at the time of writing) is the debate over what qualifies as a sandwich: Many are willing to permit hot dogs as sandwiches, but then it seems to immediately follow that tacos and wraps should be as well. Indeed, there is a deep tension at what we are intuitively comfortable labeling a sandwich. "Open-faced" sandwiches exist, but then does that make pizza a sandwich? What are the fundamental qualities separating a burrito from a hamburger?

These are the questions this work is interested in addressing. Towards that end, we build upon extant qualitative descriptions to develop a rigorous mathematical model of the problem of food identification. What follows is an overview of the qualitative model (the so-called "Cube Rule"), before describing at length the present effort to more rigorously define it.

A. The Cube Rule

The effort to illuminate these questions saw considerable progress when Twitter user `@Phosphatide` posted an illustration describing a model they dubbed the "Cube Rule" (hereafter CR)[1]. In this paradigm, the nomenclature for a given dish is completely determined by the location of the food's starch elements, as mapped onto the faces of a 3-cube (fig. 1), which are assumed to be rigid.

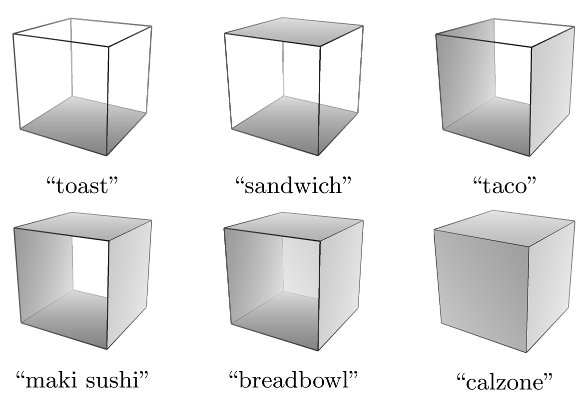

FIG. 1: The Cube Rule as originally described in a tweet by `@Phosphatide`. They are named according to their prototypical examples.

CR as a model has several strengths. In addition to generally agreeing with intuition, the model's generalization to *starch*, rather than a focus on bread in particular, allows it to describe a much more complete set of observed configurations. In the original bready conception of the question, for example, sushi would be wholly excluded. Its inclusion will later provide significant analytical utility.

To make CR rigorous, we desire a mathematical prescription which can adequately describe all of the configurations depicted in fig. 1 as elements of some as-yet-undetermined space. Clearly, this prescription should be invariant under certain group symmetries (SO(3) at least), and the absence of a configuration between toast and sandwich should be either remedied or explained[1].

II. THE TWO-DIMENSIONAL TOY MODEL

En route to a full mathematical description of CR, we first restrict ourselves to a two-dimensional version, where we consider foods as occupying sides of a square. In this model, we take as our basis set $\mathcal{B} = \{b_1, b_2, b_3, b_4\}$, for successive sides of a square, b_n (fig. 2).

$\mathcal{B}$, together with a ladder operator $\hat{A}$, form a complete algebra we can use to construct any configuration of starchy food. The action of $\hat{A}$ on a basis element b_n is defined like so:

$$\hat{A}^k b_n := (-1)^{k+1} b_{n+k \pmod 4}. \tag{1}$$

With this definition of $\hat{A}$, we have that a given b_n will return to itself again after an application of *eight* iterations of the ladder operator ($\hat{A}^8 b_n = b_n$). Thus, if $\hat{A}$ is interpreted as imparting a phase of $\pi/4$ on a b_n, we have that $\mathcal{B}$ has the symmetry of a Clifford algebra, and therefore the b_n are manifestly spinors.

In particular, b_1 and b_2 are the spin-up and spin-down pseudovectors respectively, while b_3 and b_4 have an im-

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[1] It will be shown that a single slice of certain varieties of pie are of this configuration, and *not* an example of toast, as claimed in [2]

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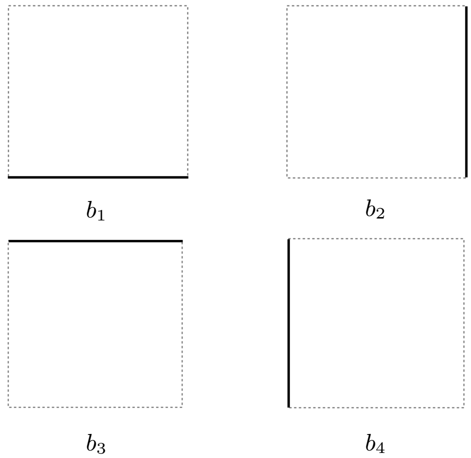


FIG. 2: Visualization of the basis set of starch positions $\mathcal{B} = \{b_1, b_2, b_3, b_4\}$. Not pictured is the implicit null element.

parted phase of $\pi/4$:

$$\begin{aligned} b_1 &\equiv \left|\uparrow\right\rangle, \\ b_2 &\equiv \left|\downarrow\right\rangle, \\ b_3 &\equiv e^{i\pi/4}\left|\uparrow\right\rangle, \\ b_4 &\equiv e^{i\pi/4}\left|\downarrow\right\rangle, \end{aligned}$$

$\hat{A}$ can now be written explicitly:

$$\hat{A} = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix} \tag{2}$$

$$= \frac{e^{i\pi/4}}{\sqrt{2}}\left(\sigma_x - \sigma_y\right), \tag{3}$$

for Pauli spin matrices σ_x, σ_y. Now each of the 4 basis starch locations in $\mathcal{B}$ can be interpreted as each being a spin-up or a spin-down spinor. The various configurations (sandwich, taco, etc...) are now superpositions of these basis states.

It is the symmetry identified by CR that allows us to make this interpretation; were it more appropriate to map starch locations to some other shape, spin formalism might not apply. We can now identify some of the more recognizable configurations[2]:

$$\text{sandwich} : \left|{}^2S\right\rangle = \left(1 + \hat{A}^2\right)\left|\uparrow\right\rangle \tag{4}$$

$$\text{taco} : \left|{}^2T\right\rangle = \left|\uparrow\right\rangle + \left(1 + \hat{A}^2\right)\left|\downarrow\right\rangle \tag{5}$$

$$\text{maki} : \left|{}^2M\right\rangle = \left(1 + \hat{A}^2\right)\left(\left|\uparrow\right\rangle + \left|\downarrow\right\rangle\right) \tag{6}$$

(The prefix superscript ${}^2X$ denotes that we are explicitly working with the 2-dimensional case. This will be dropped in the full treatment.)

[2] By convention, configurations are given with respect to positive spin up. Technically, these could be written with respect to any of the b_n.

The final state, the "makion" will be of interest to us when we extend CR to 3 dimensions. Each side of the 3-cube is constructed of makions.

Immediately, we find interesting features in the model. For instance, the number of pieces of structural starch is given by the magnitudes of the configuration states:

$$\begin{aligned} \left\langle {}^2S \middle| {}^2S \right\rangle &= 2, \\ \left\langle {}^2T \middle| {}^2T \right\rangle &= 3, \\ \left\langle {}^2M \middle| {}^2M \right\rangle &= 4, \text{etc...} \end{aligned}$$

So, we have now shown that the 2-D case is nicely described by pseudovectors in a Hilbert space, whose group symmetry is identical to that of spinors. We will now extend this model to 3 dimensions, and finally describe the full self-consistent rigorous CR treatment.

III. A SELF-CONSISTENT MODEL OF STARCH PLACEMENT

For the three-dimensional case, we take as our basis set a two-dimensional makion rotated onto each of the faces of one corner a 3 cube (fig. 3).

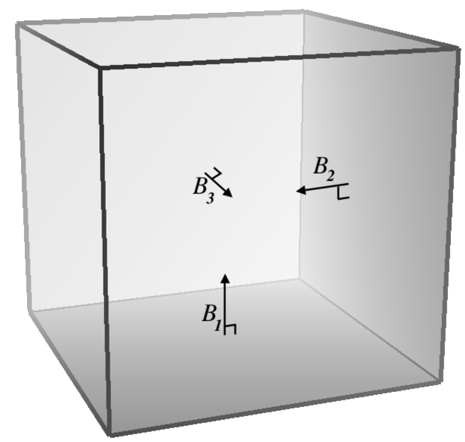


FIG. 3: The basis set for the 3-dimensional set.

This is effectively the same as the two-dimensional case, with an extra element. The other elements are achieved by rotating each of the basis vectors by $\pi/4$.

So where the two-dimensional vectors were spinors, we can make an analogue in three-dimensions. We identify the orthonormal vectors

$$\begin{aligned} B_1 &\equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \left|\Uparrow\right\rangle, \\ B_2 &\equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \left|\Rightarrow\right\rangle, \\ B_3 &\equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \left|\Downarrow\right\rangle, \end{aligned}$$

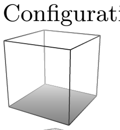
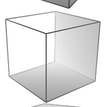
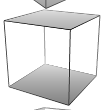
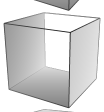
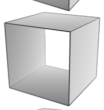
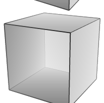
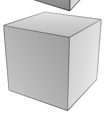
where each of the B_n are understood to be makions with area unit vectors pointing in the positive directions in their respective planes.

The ladder operator, $\hat{\mathbb{A}}$ is a now tensor whose entries are extended to the quaternions, such that twelve applications of $\hat{\mathbb{A}}$ will return a B_n to itself after being rotated through the entire cube:

$$\hat{\mathbb{A}}^k B_n := (-1)^k B_{n+k \pmod 6}. \tag{7}$$

We can now give precise mathematical definitions of the configurations given in fig. 1. Table 1 provides a description of the menu of some starch configurations as written in the original tweet, along with their mathematical properties.

TABLE I: Menu of starch configurations. This is not exhaustive.

Configuration	Name	Expression	Examples
	toast	$\lvert\mathbf{B}\rangle = \lvert\Uparrow\rangle$	toast, pizza, nigiri
	pie slice	$\lvert\mathbf{L}\rangle = \lvert\Uparrow\rangle + \lvert\Downarrow\rangle$	pie slice
	sandwich	$\lvert\mathbf{S}\rangle = (1 + \hat{\mathbb{A}}^2)\lvert\Uparrow\rangle$	PB&J, torta, hamburger
	taco	$\lvert\mathbf{T}\rangle = \lvert\mathbf{S}\rangle + \lvert\Downarrow\rangle$	taco, hot dog, crusted pie slice
	maki	$\lvert\mathbf{M}\rangle = \sum_n^6 \hat{\mathbb{A}}^{2n}\lvert\Uparrow\rangle$	maki, taquito, crepe
	breadbowl	$\lvert\mathbf{R}\rangle = \lvert\mathbf{M}\rangle + \lvert\Rightarrow\rangle$	crustless pie, gyro, deep-dish pizza
	wrap	$\lvert\mathbf{W}\rangle = \lvert\mathbf{M}\rangle + \hat{\mathbb{A}}\lvert\mathbf{S}\rangle$	burrito, baked brie, calzone

A. The Existence of Seventh and Eighth Configurations

One of the original motivations for this work was to address the lack of a listed configuration between toast and sandwich. Some starch configuration, of the form $\lvert\Uparrow\rangle + \lvert\Downarrow\rangle$ should exist here. That it doesn't is surprising. Is this simply never observed in nature?

We can investigate this problem by looking for this configuration in the theory as it stands. If we consider the breadbowl as representing a crustless pie, (pumpkin, key lime, meringue, etc...), we can define the "slice operator" to remove exactly the configuration we're looking for.

Defining

$$\mathbb{S} \equiv \frac{1}{1+i}\mathbb{I}, \tag{8}$$

we have

$$\mathbb{S}\lvert\mathbf{R}\rangle = \lvert\Uparrow\rangle + \lvert\Downarrow\rangle\,. \tag{9}$$

Thus, the slice we were looking for is contained in the breadbowl. Importantly, we can *reconstruct* the breadbowl by rotating the slice around only a single Euler angle.

When we identify the breadbowl with a crustless pie, we can see that the slice we have constructed above is indeed a slice of crustless pie, and *not* a piece of toast, as claimed in [2]. Indeed, this agrees with intuition: Slices of pie *clearly* have a bread component orthogonal to their base. In fact, it is a configuration not yet listed in the literature.

In a similar way, we can slice the wrap (or the breadbowl again) to reveal one more basic configuration: $\lvert\Uparrow\rangle + \lvert\Downarrow\rangle + \lvert\Rightarrow\rangle$. This configuration is purely theoretical at the time of this writing. There is no known foodstuff which possesses a "corner" like this. We leave it to the experimentalists to design an experiment to detect this so-called "corner bread".

IV. DISCUSSION

We have now prescribed a self-consistent mathematical description of a model which classifies food based on their starch positions. We emphasize, however, that the theory is yet in its infancy; there are doubtless edge-cases and potential counter examples to contend with.

One of the first interesting extensions of this model is to consider the case of foods which explicitly break the symmetries this model depends on. For example, the McDonald's Big Mac contains a *third* slice of bread in the middle of the sandwich. It turns out that this model is sufficiently generalized to handle this case nicely. The magnitude of one of the sandwich's components is doubled. It does not matter which one, owing to the Big Mac's parity invariance.

A potential fruitful avenue of exploration would be to investigate composition of these vectors. The behavior of two burritos on the same plate has as-yet-unknown mathematical properties.

V. REFERENCES

[1] `@Phosphatide`. https://twitter.com/Phosphatide/status/974067376894328833. Twitter, 2018.

[2] Ryan. *The Cube Rule*. http://cuberule.com/